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European Competence Center for IT Security

Public-Key Building Blocks

Summer School on Cryptographic Hardware, Side-Channel and Fault Attacks

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1. Why do we need public-key cryptography?
2. Overview on public-key crypto schemes
3. Arithmetic
4. Open research problems

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1. **Why do we need public-key cryptography?**
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IT Security vs. Cryptography

1. IT Security ≠ Cryptography
2. but: Cryptography is an important **tool** for achieving secure IT systems

Cryptography

Symmetric Algorithms
(2000 B.C. ... 1976)

Public-key Algorithms
(Diffie/Hellman/Merkle 1976)

The Cryptographic Toolkit

Cryptographic Algorithms

Symmetric

- Stream ciphers
- Block ciphers

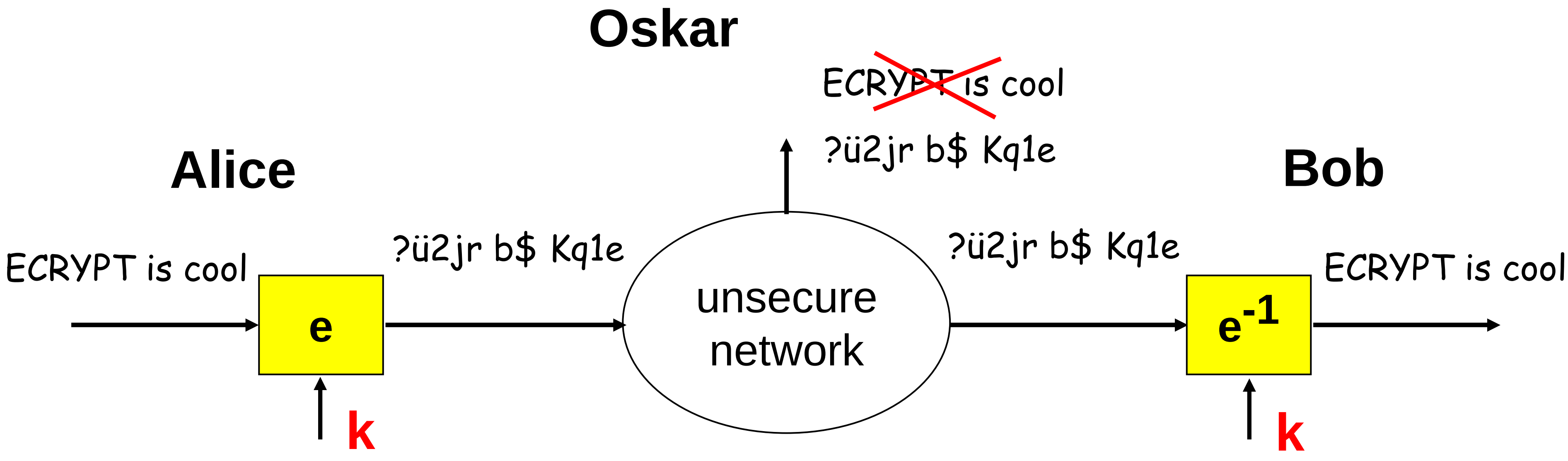
Asymmetric

- Integer Factorization (RSA...)
- Discrete Logarithm (D-H, DSA,...)
- Elliptic Curves (ECDH, ECDSA,...)

Hash Fct.

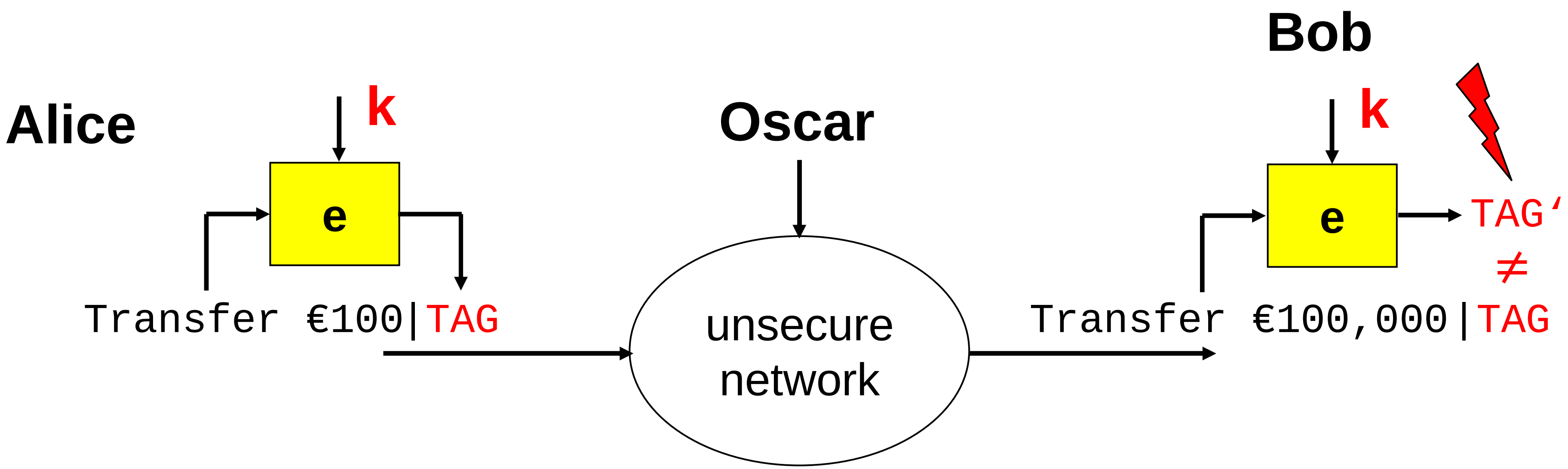
- MD5
- SHA-1
- SHA-XY
- ?

What we can do with symmetric crypto (I): Confidentiality



Encryption ensures **confidentiality** of messages

What we can do with symmetric crypto (II): Message Integrity



Message Authentication Codes (MAC) detect malicious integrity violations

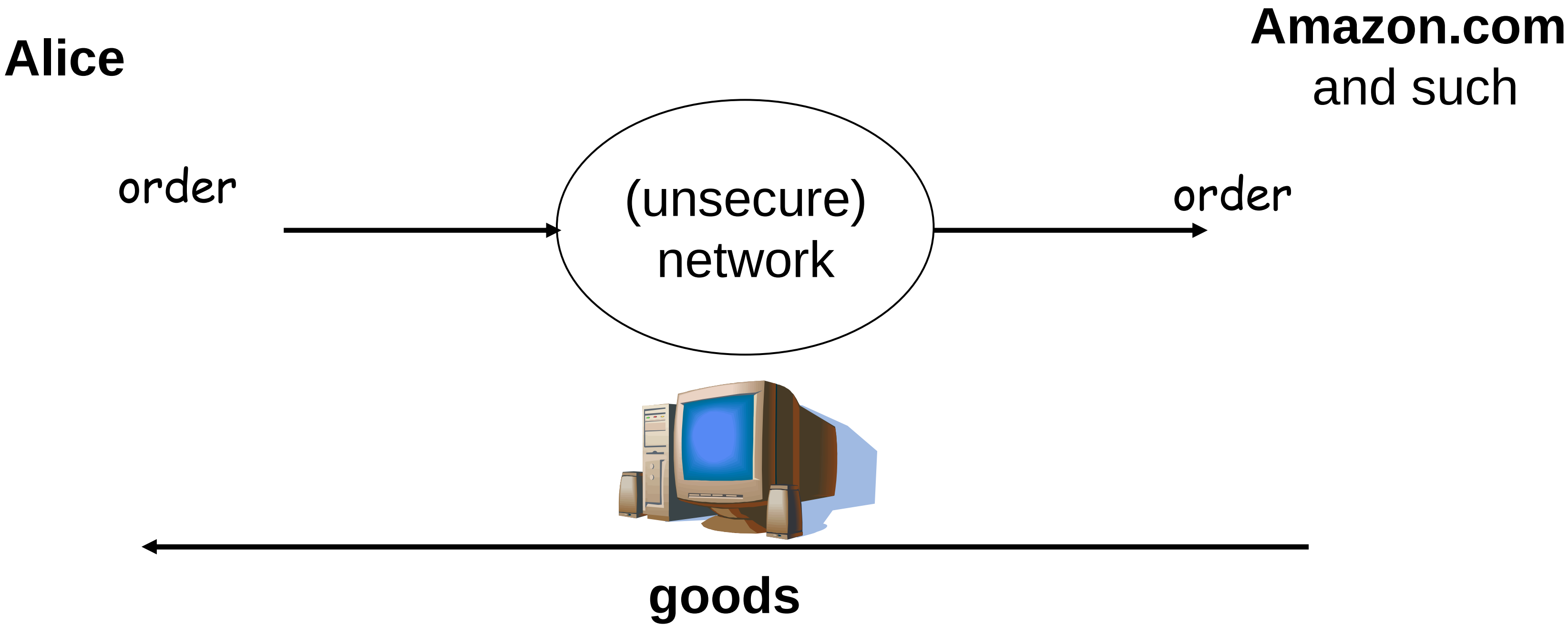
What do we need public-key (or asymmetric) cryptography for?

Two main functions:

1. Key distribution over unsecure channel
2. Digital Signatures for non-repudiation
3. [Encryption]

Rem: symmetric ciphers are still needed because public-key algorithms are awfully slow.
(Note: purely practical/engineering reason)

Non-repudiation: Why we need it



without non-repudiation:

1. Alice orders at favorite eCommerce vendor
2. stuff gets delivered
3. Alice doesn‘t feel like buying: „I never ordered this“
4. vendor can not **proof** it (big monetary issue if vendor = BMW.com)

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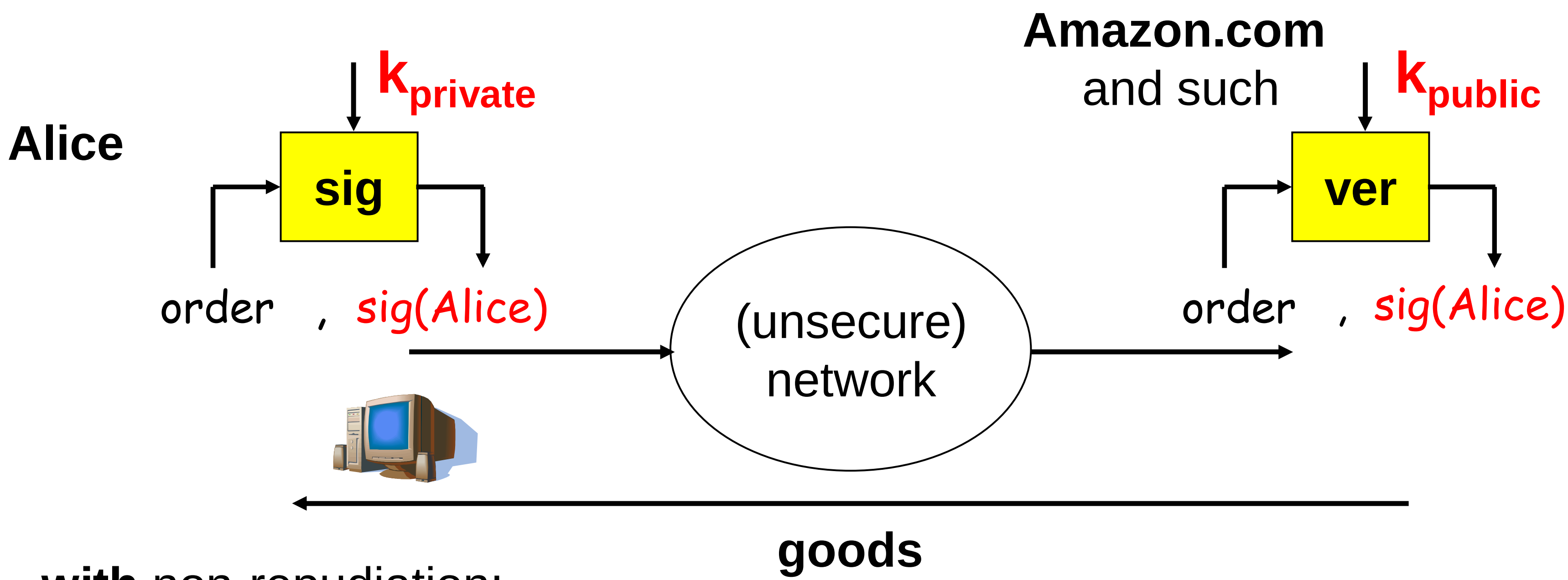
Non-repudiation with Digital Signature



with non-repudiation:

1. Alice orders at favorite eCommerce vendor
2. stuff gets delivered
3. Alice doesn‘t feel like buying: „I never ordered this“
4. vendor sues Alice: **proof** of order through Alice‘s signature (only Alice knows $k_{private}$, not even the vendor!)

Non-repudiation is strong point of asymmetric cryptography

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The World of Public-key Algorithms

Much fewer schemes than in the symmetric case!

Public-key Schemes

Established Algorithms

1. Integer factorization family
2. Discrete log family
3. Elliptic curve family

Not-so established Alg.

- lattice-based (NTRU)
- high-field equations
- code-based (McEliece)
- …

Established public-key algorithms

The 3 families of algorithms of practical relevance:

Integer Factorization

Ex: RSA, Rabin, ...

Operands: 1024 – 4096 bits

Discrete Logarithm

Ex: Diffie-Hellman, DSA, ...

Operands: 1024 – 4096 bits

Elliptic Curves (ECC)

Ex: EC Diffie-Hellman, ECDSA, ...

Operands: 160 – 256 Bits

Observation: All asymm. algorithms require heavy computation

How many key bits do I need?

symmetric	*ECC*	*RSA, DL*	*comment*
64 bit	128 bit	≈ 700 bit	only short term security (breakable with some effort)
80 bit	160 bit	≈ 1024 bit	medium term security (excl. government attacks)
128 bit	256 bit	≈ 2048-3072 bits	long term security (not assuming quantum computers)

- Exact complexity of RSA (factorization) and DL (index-calculus) attacks is hard to determine
- Quantum computer would probably be the death of ECC, RSA & DL (but don‘t hold your breath – at least a few decades away)

Arithmetic requirements of PK algorithms

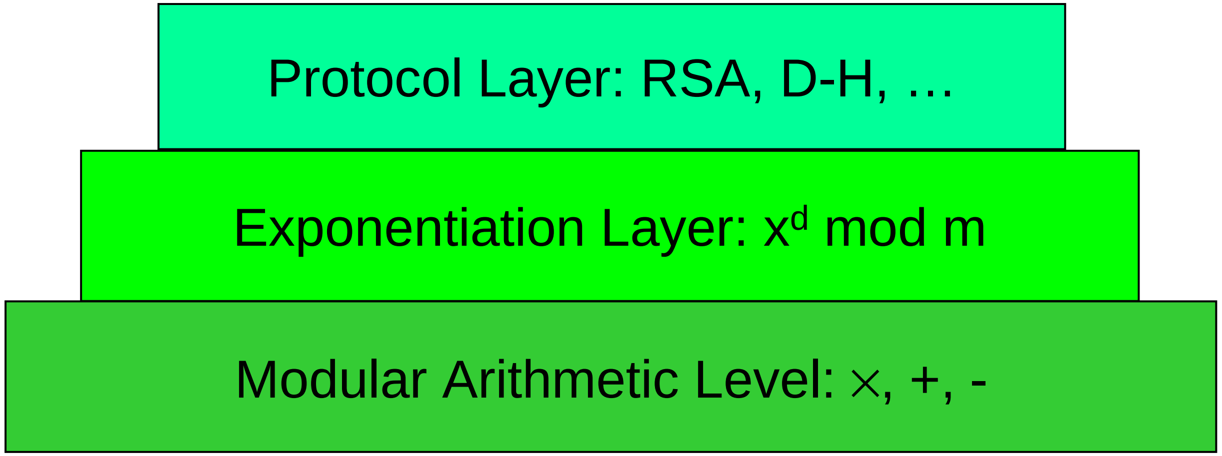
Algorithm	*typ. operand length (mult)*	*# multipl. / group op*	*# multipl. / crypto fct.*
RSA	1024 bit	1	17 (verify) ≈ 1300 (sign)
Discrete log	1024 bit	1	≈ 200
Elliptic Curves	160 bit	≈ 10	≈ 2000

Observations:

- RSA is „best“ for signature verification
- ECC is „best“ for signature generation
- ECC has other advantages (bandwidth etc)
- RSA by far outnumbers ECC implementations in practice (but ECC is slowly catching up)!

Hierarchical System Design of RSA and DL Engines

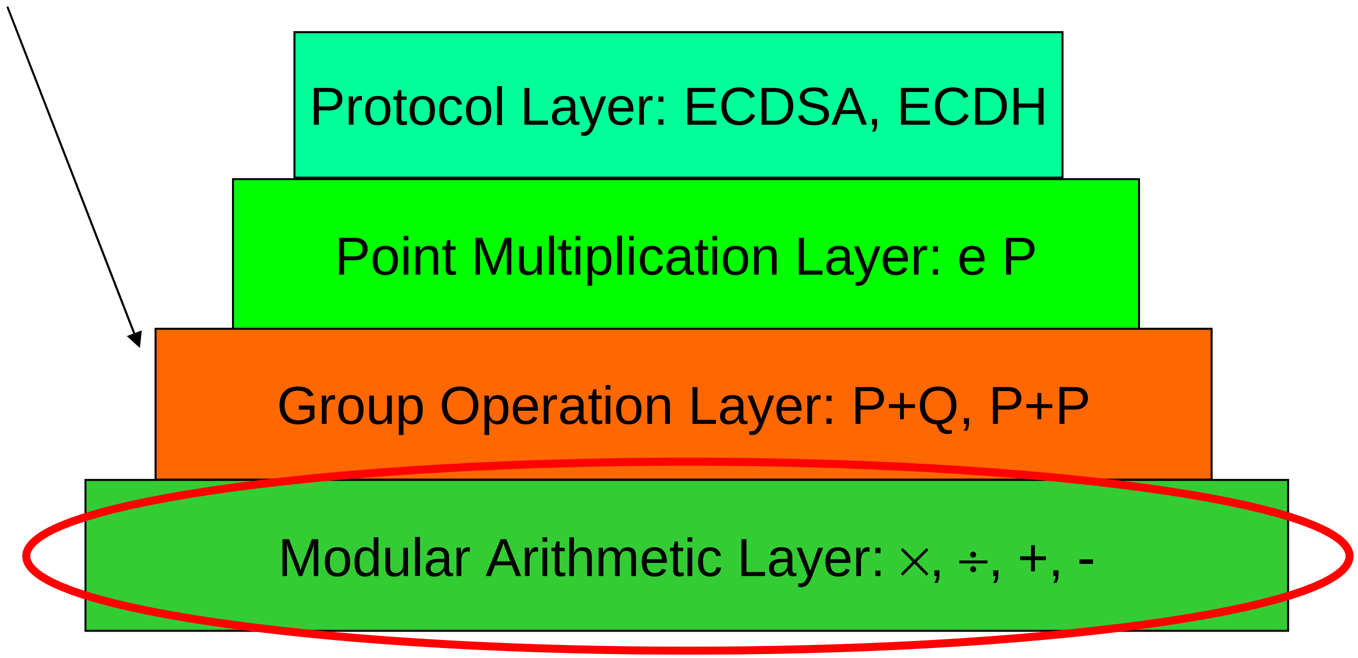
RSA, DL engines are mainly exponentiation units



Rem: > 90% of computation time is spent on modular multiplication

Hierarchical System Design of ECC Engines

Group Operation Layer added



Rem: Still > 90% of computation time is spent on modular multiplication (and on inversion, if affine coordinates are used)

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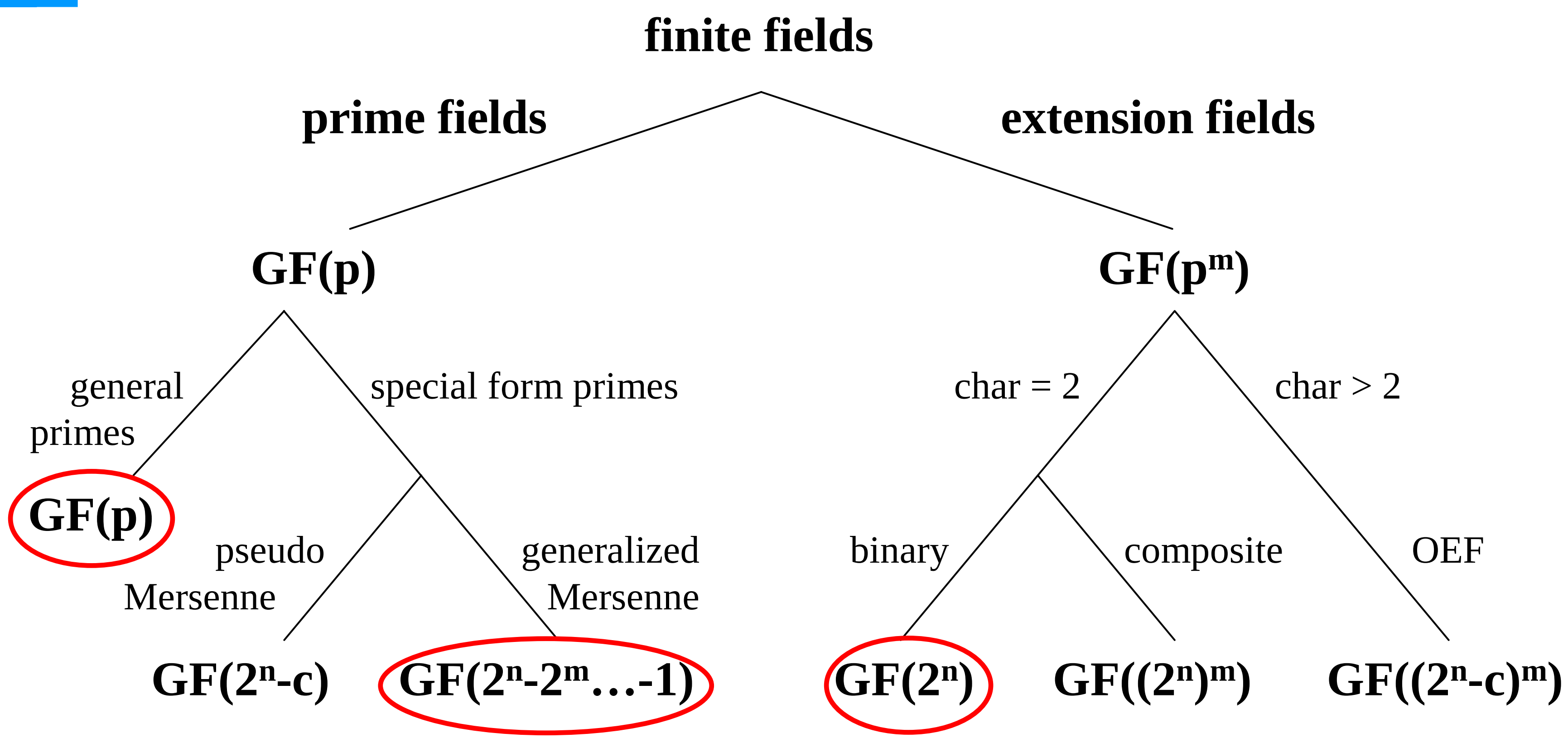
Arithmetic proposed for use in public-key schemes



- DL, ECC are based on finite fields (= Galois fields)
- RSA arithmetic similar to GF(p) arithmetic

Prime Fields GF(p)

Relevance

DL: GF(p) is the only field type used in practice

ECC: GF(p) somewhat more popular than $GF(2^m)$

RSA: modular m=p q arithmetic, but algorithms almost identical

$\Rightarrow$ GF(p) is most important field in practice

Basics about GF(p) arithmetic

- addition, subtraction is cheap
- inversion is much slower than multiplication
  (hence, ECC is often used with projective coordinates)
- "Remaining" problem:

Efficient modular multiplication methods for
160-4096 bit numbers?

Prime Fields GF(p): Software I

Ex: A, B ϵ GF(p), $p < 2^{4096}$, word size $w = 32$

Element representation (on 32 bit machine):

$A = a_{127}\, 2^{127\times 32} + \ldots + a_1\, 2^{32} + a_0 \quad , a_i \in \{0,1, \ldots , 2^{32}-1\}$

$B = b_{127}\, 2^{127\times 32} + \ldots + b_1\, 2^{32} + b_0 \quad , b_i \in \{0,1, \ldots , 2^{32}-1\}$

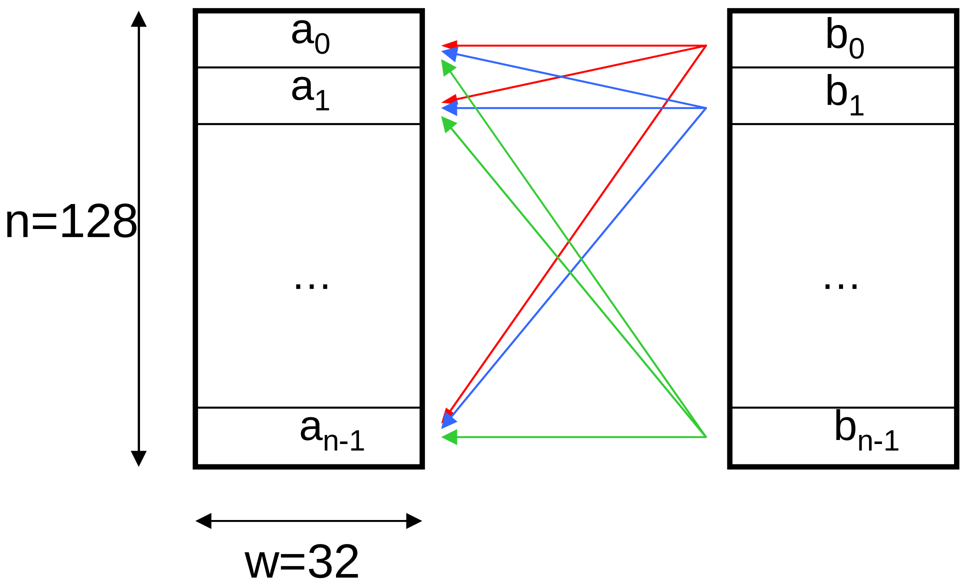
Goal: Compute A x B mod p efficiently

For the beginning, a simple approach:

1. **Step: Multi-precision multiplication**
2. **Step: Modular reduction**

1. Step: Multi-precision Multiplication

$$C' = A \times B$$



Complexity

n^2 integer multiplications

(Ex: $n^2 = 128^2 = 16{,}384$ int. mult.)

Remark

Quadratic complexity can be reduced to $n^{1.58}$ using Karatsuba‘s algorithm

2. Step: Modular Reduction

$$C \equiv C' = A * B \bmod p$$

1. (naive) approach: long division of C' by p
2. (better) approach: fast modulo reduction techniques, avoiding division:
   2.1. Montgomery
   2.2. Barrett
   2.3. Sedlack
   2.4. …

reduction compl. $\approx n^2$ integer mult.

Note: fast mult. methods à la Karatsuba not applicable!

$\Rightarrow$ **total modular mult. compl. $\approx$ 2 n^2 integer mult.**

Rem: Multi-precision mult (Step 1) and modular reduction (Step 2) are often interleaved. Complexity does not change.

Montgomery Reduction in Hardware I

p is an n-bit number: $n = \lceil \log_2 p \rceil$

Idea: Compute *n* inner products in parallel

Best studied architecture: Montgomery multiplication

Input: A, B, where $A = \sum_{i=0}^{n+2} a_i 2^i$, $B = \sum_{i=0}^{n+1} b_i 2^i$
Output: $A \cdot B \bmod N$

1. $R_0 = 0$
2. for $i = 0$ to $n + 2$ do
3. $q_i = R_i(0)$
4. $R_{i+1} = (R_i + a_i \cdot B + q_i \cdot N)/2 \qquad (\star)$

time complexity (radix 2):
n clock cycles

time complexity (radix r):
n/r clock cycles

$\Rightarrow$ *O*(*n*) times faster than software (which has n^2)

area complexity:
cnst * *n* gates

Montgomery Reduction in Hardware II

Remarks

1. modular reduction is reduced to addition of long numbers:

$$R_{i+1} = (R_i + a_i B + q_i N) / 2$$

2. Use redundant representation or systolic array to avoid long carry chains

3. Division only by 2 (or 2^r) $\Rightarrow$ only right shifts

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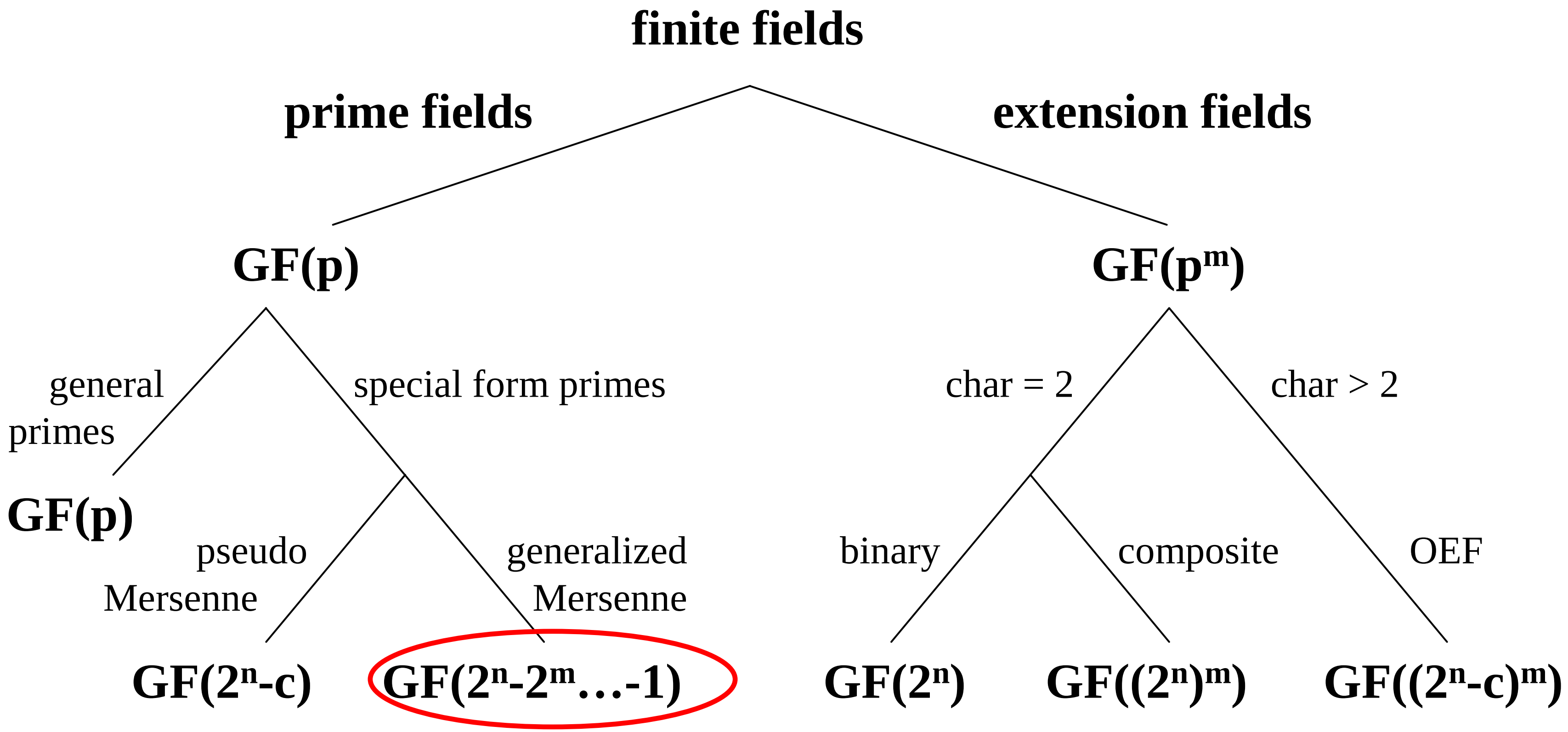
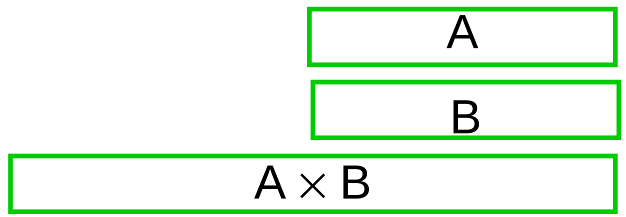
Generalized Mersenne Primes



very attractive for ECC!

Generalized Mersenne Primes: Example

Prime: $p = 2^{192} - 2^{64} - 1$, $w = 64$



$A = a_2 2^{128} + a_1 2^{64} + a_0$

$B = b_2 2^{128} + b_1 2^{64} + b_0$

$A \times B = c_5 2^{320} + c_4 2^{256} + c_3 2^{192} + c_2 2^{128} + c_1 2^{64} + c_0$

Reduction equations

$2^{320} \equiv 2^{192} + 2^{128} \mod p$

$2^{256} \equiv 2^{128} + 2^{64} \mod p$

$2^{192} \equiv 2^{64} + 1 \mod p$

$A \times B \equiv c_5 (2^{192} + 2^{128}) + c_4 (2^{128} + 2^{64}) + c_3 (2^{64} + 1) + c_2 2^{128} + c_1 2^{64} + c_0 \mod p$

$A \times B \equiv [c_5 + c_4 + c_2] 2^{128} + [c_5 + c_4 + c_3 + c_1] 2^{64} + [c_5 + c_3 + c_0] \mod p$

Modular reduction is realized with a few additions!
(no multiplications, no inversions)

Generalized Mersenne Primes and ECC

- Specific primes recommended by NIST: 192, 224, 256, 384, 521 bit
- Reduction requires no multiplication, only additions
- Roughly **twice as fast** as modular multipl. with general primes
- Very popular for ECC in practice

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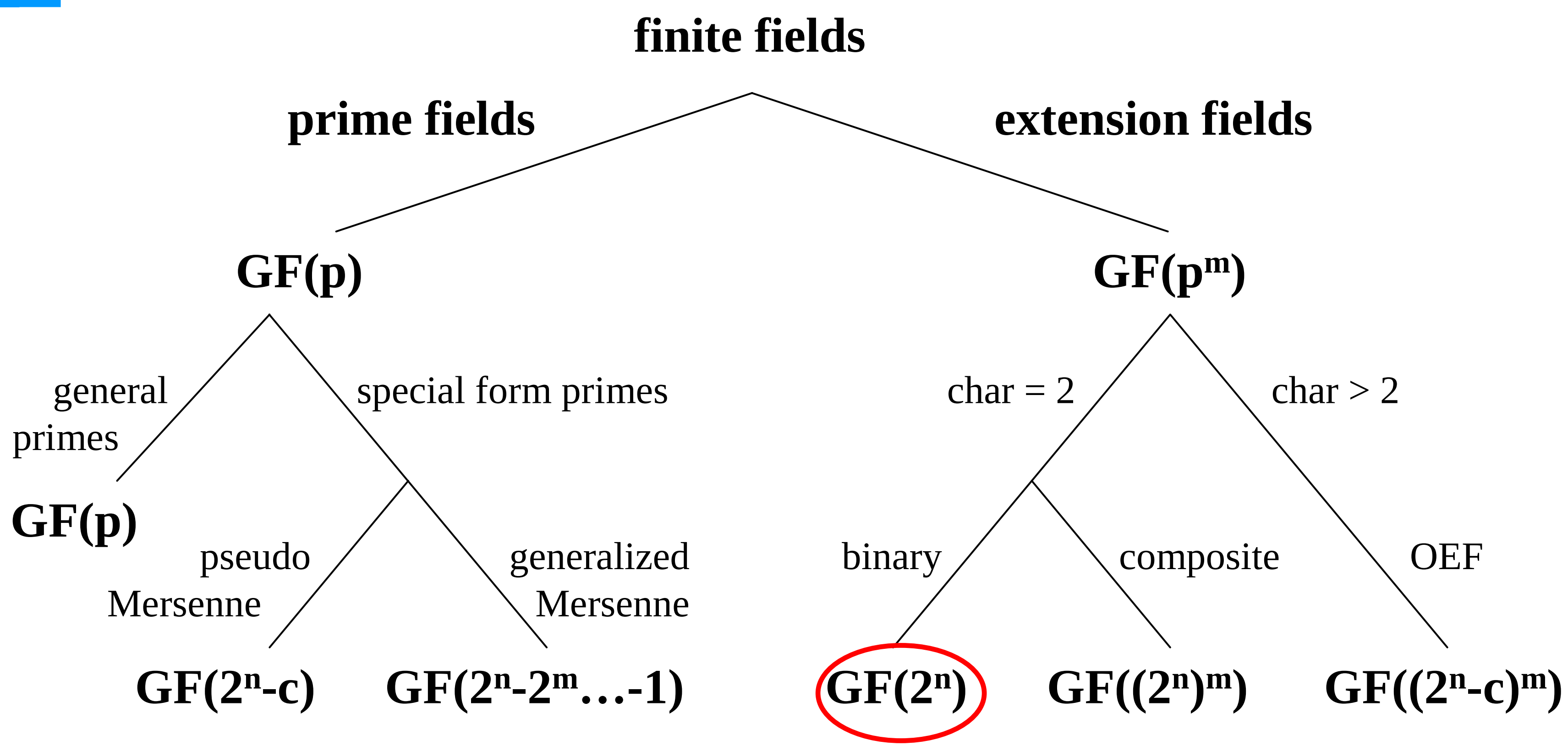
Binary Fields GF(2^m)



Multiplication is the most critical operation in most applications

Basic Facts about Binary Fields $GF(2^m)$

1. main application in modern PK: **Elliptic Curve Cryptosystems**
2. also applicable for DL, but index-calculs attack works somewhat better in $GF(2^m)^*$ than in $GF(p)^*$

   ⇒ **rarely used anymore for DL problems**

3. **very well studied** compared to other extension fields since 1960s (applications in channel coding for early space missions)
4. choice of char = 2 was traditionally driven by **hardware implementations**
5. arithmetic is greatly influenced by choice of basis
   – polynomial basis
   – normal basis
   – other (dual basis, triangular basis, …)

   polynomial basis most attractive for PK crypto in practice

A Big Question: $GF(2^m)$ vs GF(p) for ECC ?

A long story made short

1. **Software: GF(p) is somewhat faster** if carefully implemented.
   (Note that the vast majority of implementations run in software)
2. **Hardware:** $GF(2^m)$ has a much **better time-area product** than GF(p)
3. It is believed that the **patent situation** is less messy in the GF(p) case
4. There is a trend that **GF(p) is more common in practice**
   (due to national standards in the US and Europe & patent situation)
5. $GF(2^m)$ in hardware is highly attractive for **light-weight crypto**
   (RFID and such)

GF(2^m) Multipliers for Hardware

- many proposed architectures
- classification according to time-area trade-off

smaller & slower (↓)

architecture	#clocks (time)	#gates (area)	m	Remarks
bit parallel	1	O(m^2)	any	usually „too big“ for PK crypto
hybrid	m/D	O(mD)	D|m	can lead to weak ECC
digit serial	m/D	O(mD)	any	digit size D allows scaling
bit serial	m	O(m)	any	classical arch.
super serial	ms	O(m/s)	any	SW-like, only if RAM cheap

Main relevance in cryptography: **bit serial and digit serial**

Bit Serial Multiplication

Polynomial-basis multiplication

$$A \times B = (a_0 + \ldots + a_{m-1} x^{m-1}) \times (b_0 + \ldots + b_{m-1} x^{m-1}) \bmod P(x)$$

where $a_i, b_i \in GF(2)$

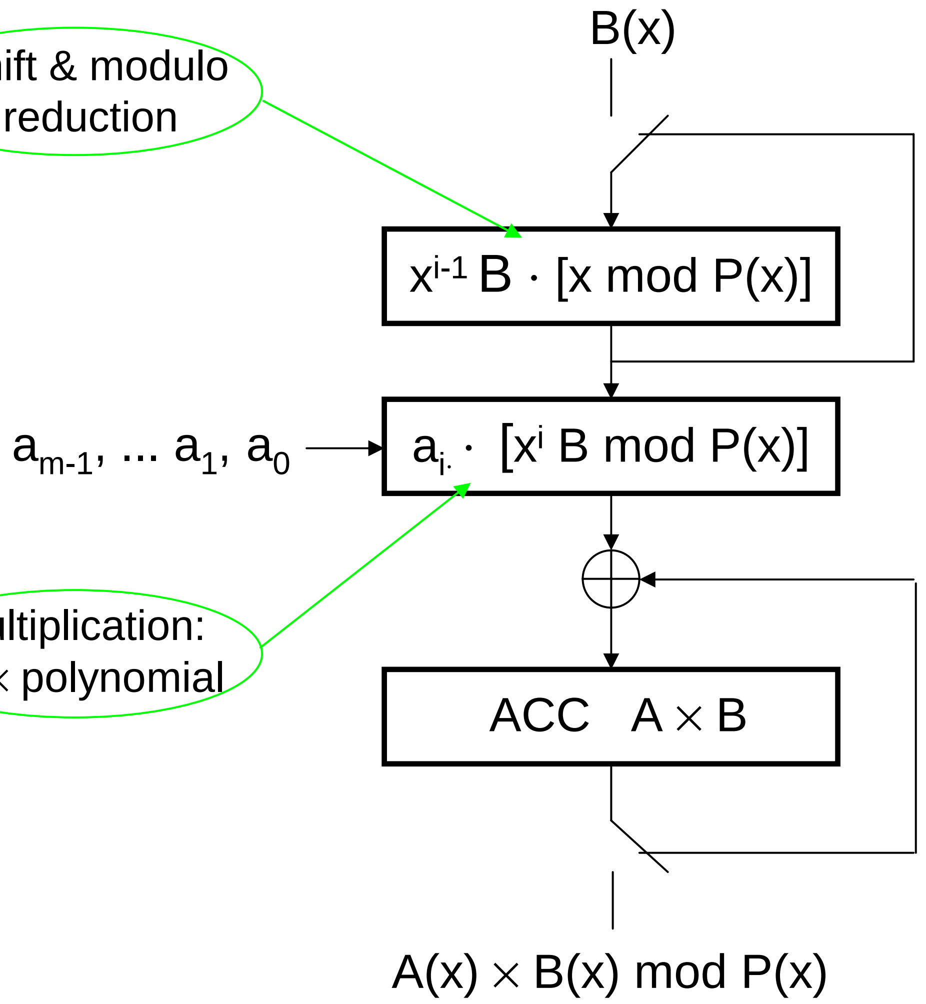
In practice: *P*(*x*) is almost always trinomial or pentanomial

Two traditional architectures

- least significant bit-first (LSB) multiplier
- most significant bit-first (MSB) multiplier

Least Significant Bit GF(2^m) Multiplier

$$A \times B = a_0\, B(x) + a_1 \cdot [x\, B \bmod P(x)] + \dots + a_{m-1} \cdot [x\, x^{m-2}\, B \bmod P(x)]$$



Time: m clock cycles

Area: cnst × m gates (cnst small)

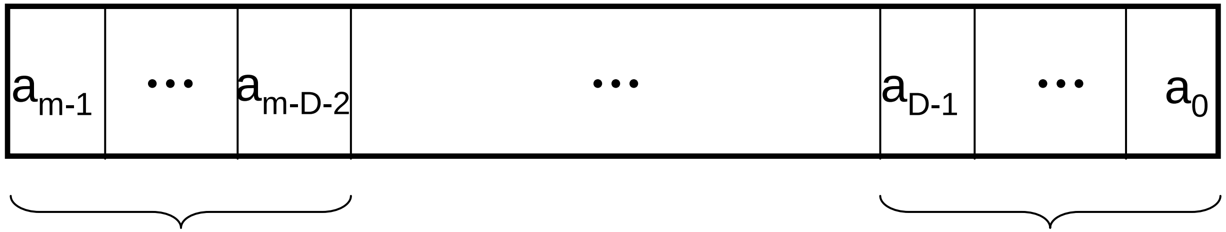
Digit Multipliers for $GF(2^m)$

1. generalization of bit-serial multipliers
2. fundamental idea: **process *D* > 1 bit** at a time
3. works for any *m*
4. trades space for speed: **faster but larger than bit-serial** architectures
5. **time-area product is constant** (at least under big-O notation)
6. LSD (least significant digit) and MSD (most significant digit) are possible

Least Significant Digit Architecture

Idea: Break A(x) down into s digit polynomials

$$A(x) = a_{m-1}x^{m-1} + \ldots + a_1 x + a_0 \quad , \quad a_i \in GF(2)$$



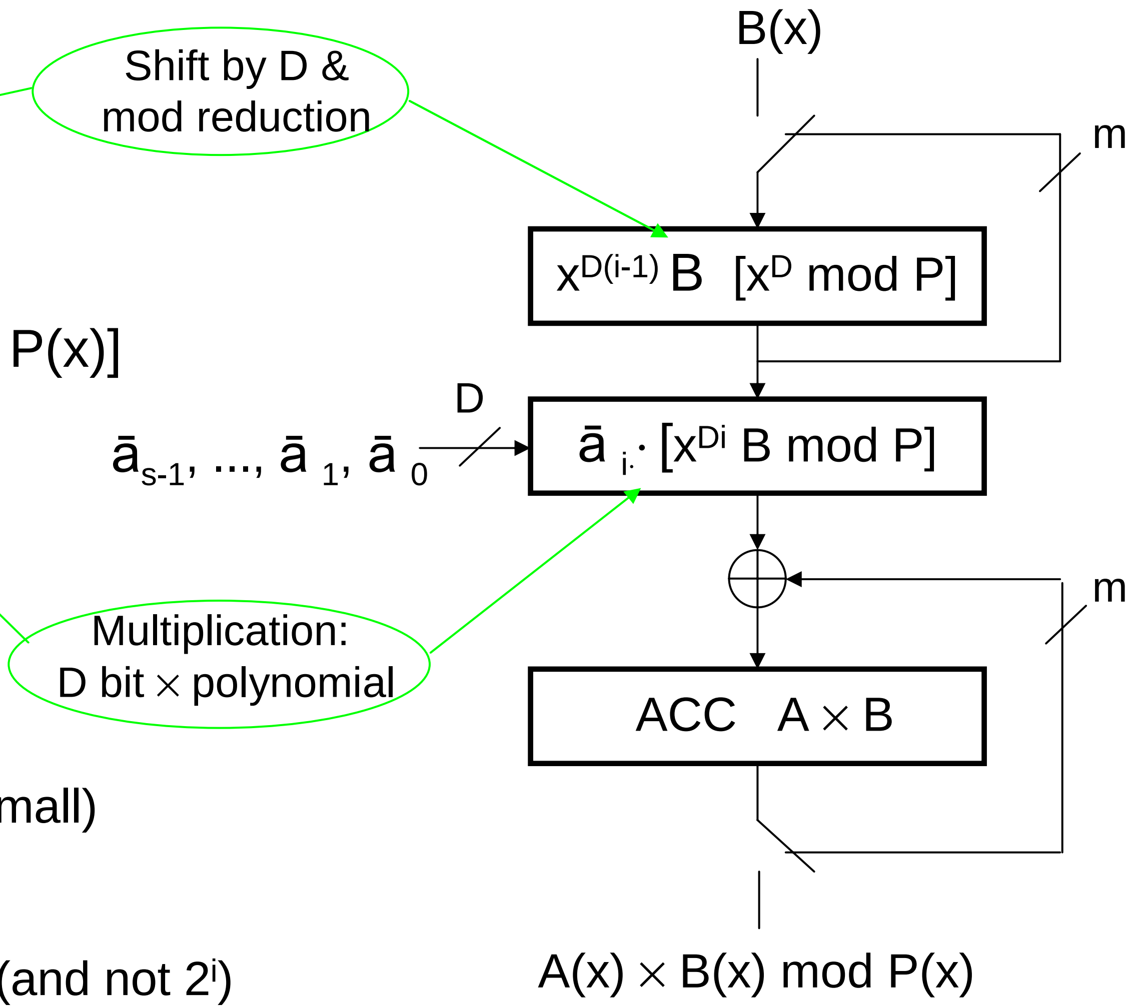
$s = \lceil m/D \rceil$ $\qquad \bar{a}_{s-1} \qquad \bar{a}_0$

$$A(x^D) = \bar{a}_{s-1}x^{(s-1)D} + \ldots + \bar{a}_1 x^D + \bar{a}_0$$

where $\bar{a}_i = \bar{a}_i(x) = a_{i,D-1}x^{D-1} + \ldots + a_{i,1}x + a_{i,o} \quad , a_{i,j} \in GF(2)$

Least Significant Digit GF(2^m) Multiplier

$$A \times B = \bar{a}_0\, B(x)$$
$$+\ \bar{a}_0 \cdot\ [x^D\, B \bmod P(x)]$$
$$+\quad \cdots$$
$$+\ \bar{a}_{m-1} \cdot\ [x^D\, x^{Dm-2}\, B \bmod P(x)]$$



Time: ≈ m/D clock cycles

Area: cnst × m D gates (cnst small)

Watch out: optimum $D = 2^i - 1$ (and not 2^i)

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Challenges in Applied Public Key Cryptography

1. Highly efficient implementations of established alg. (RSA, DL, ECC) for light-weight crypto

2. New PK algorithms with low implementation complexities

3. $GF(p^m)$ ("OEF") has nice implementation properties in software: Security of such fields for discrete log and ECC?

4. Special-purpose hardware for PK cryptanalysis

5. Better understanding of side channel and tamper resistance

Related Workshops

RFIDSec
July 2006, Graz

CHES – Cryptographic Hardware and Embedded Systems (+ FDTC)
October 2006, Yokohama

escar – Embedded Security in Cars
November 2006, Berlin